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A stochastic frontier model based on Rayleigh distribution

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In this paper, we present a closed formula for calculating the density of the composed error in a stochastic frontier model, having supposed that technical inefficiency components follow a Rayleigh probability distribution. Moreover, by using a Monte Carlo procedure, we analyze the properties of Maximum Likelihood and Method of Moments estimators of the disturbance terms. Then, we utilize recent historical data to judge the performance of various estimators.

keywords: Stochastic frontier analysis, Rayleigh distribution, Monte Carlo methods.

1 Introduction

The development of the stochastic frontier analysis in econometric is primarily due to Aigner et al. (1977). Traditionally, the efficiency production analysis focuses on estimating average and frontier production functions (see Farrell, 1957 or Mishra, 2007). Aigner et al. (1977) were the first to introduce additional random variables, representing noise and technical inefficiency, in the production models.

In stochastic frontier analysis literature, the authors always assume that any component of noise follows a normal distribution (Behr and Tente, 2008); so, the two sided distribution models risk factors not directly controlled by the firm. On the contrary, the distribution, followed by technical inefficiency terms, may vary in relation to the assumptions made on the model, but it is always one-tailed: this depends on the production that must lie from a same part with respect to the frontier. Aigner et al. (1977)

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modelled the technical inefficiency by a half-normal distribution. More in detail, we may utilize such a distribution when the disturbances are, for the most part, close to zero. In the same article, the authors also introduced the exponential model: this last approach, such as the half-normal assumption, subsumes that the probability density function, of the technical inefficiency, is strictly positive at origin. Furthermore, Stevenson (1980) developed a model, in which the one-tailed terms followed a shifted half-normal distribution: practically, he considered a truncated normal distribution. Finally, Greene (1990) developed a new model involving the Gamma distribution: it is more flexible with respect to others, but, in this case, the composed error density is not calculable in a closed form.

In this work, we suppose that the technical inefficiency components follow a Rayleigh distribution. The outline of this paper is as follows: in Section 2 we present a closed formula for calculating the density of the composed error. Moreover, in Section 3 we investigate bias and variance of Maximum Likelihood (ML) estimators of composed error terms, by a Monte Carlo analysis. Furthermore, in Section 4 we introduce the Method of Moments (MOM) estimators and discuss their properties. Finally, in Section 5 we compare the performance of the estimators by analysing data in Baten et al. (2009).

2 The Rayleigh model for stochastic frontier analysis

We consider the production function:

$$y_i = g(x_i, b)\exp(v_i)\exp(-u_i), \quad (i \in \{1, 2, ..., I\})$$

which, in logarithmic form, is:

$$\log(y_i) = \log(g(x_i, b)) + v_i - u_i,$$

where, for any $i \in \{1, 2, ..., I\}$, y_i is the output of firm i, x_i is a vector of K inputs and b is a vector of parameters. Furthermore, v and u are $I-$dimensional random variables, representing, respectively, a symmetric disturbance and technical inefficiency. Therefore, we make the following assumptions:

1. v and u are uncorrelated.
2. For any $i \in \{1, 2, ..., I\}$, v_i follows a normal distribution with mean 0 and variance σ^2.
3. For any $i \in \{1, 2, ..., I\}$, $u_i \geq 0$.
4. For any $(i \times j) \in \{1, 2, ..., I\} \times \{1, 2, ..., I\}$, with $i \neq j$, $corr(v_i, v_j) = corr(u_i, u_j) = 0$.
5. Any component of u follows a Rayleigh distribution with parameter λ.

So, we can write

$$f_u(u; \lambda) = \frac{u}{\lambda^2} e^{-\frac{u^2}{2\lambda^2}}, \quad u \geq 0, \quad \lambda > 0$$

and

$$f_v(v;\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{v^2}{2\sigma^2}}, \qquad \sigma > 0.$$

Now, we let $\varepsilon := -u + v$; so, we can calculate the joint density of ε and u :

$$f_u(u;\lambda) f_v\left(u+\varepsilon;\sigma^2\right) = \frac{u}{\lambda^2} e^{-\frac{u^2}{2\lambda^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(u+\varepsilon)^2}{2\sigma^2}} =$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \frac{u}{\lambda^2} e^{-\frac{(\lambda^2+\sigma^2)u^2+\lambda^2\varepsilon^2+2\lambda^2 u\varepsilon}{2\lambda^2\sigma^2}} =$$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\varepsilon^2}{2\sigma^2}} \frac{u}{\lambda^2} e^{-\frac{u^2}{2}\left(\frac{1}{\lambda^2}+\frac{1}{\sigma^2}\right)} e^{-\frac{u\varepsilon}{\sigma^2}}.$$

Therefore, the probability density of the composed error is:

$$\int_0^{+\infty} f_u(u;\lambda) f_v\left(u+\varepsilon;\sigma^2\right) du = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\varepsilon^2}{2\sigma^2}} \int_0^{+\infty} \frac{u}{\lambda^2} e^{-\frac{u^2}{2}\left(\frac{1}{\lambda^2}+\frac{1}{\sigma^2}\right)} e^{-\frac{u\varepsilon}{\sigma^2}} du =$$

$$= \frac{\sigma}{\lambda^2+\sigma^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{\varepsilon^2}{2\sigma^2}} \int_0^{+\infty} \frac{u}{\frac{\lambda^2\sigma^2}{\lambda^2+\sigma^2}} e^{-\frac{u^2}{2\left(\frac{\lambda^2\sigma^2}{\lambda^2+\sigma^2}\right)}} e^{-\frac{u\varepsilon}{\sigma^2}} du.$$

It is know that, if γ and t are two real numbers, then

$$\int_0^{+\infty} \frac{u}{\gamma^2} e^{-\frac{u^2}{2\gamma^2}} e^{tu} du = 1 + \gamma t e^{\gamma^2 t^2/2} \sqrt{\frac{\pi}{2}} \left(erf\left(\frac{\gamma t}{\sqrt{2}}\right) + 1 \right),$$

where erf is the error function. Observe that this last integral is the moment generating function of a Rayleigh distribution with parameter γ (Papoulis, 1984). Finally, by letting

$$\gamma^2 = \frac{\lambda^2\sigma^2}{\lambda^2+\sigma^2}$$

and

$$t = -\frac{\varepsilon}{\sigma^2},$$

we obtain the density of the composed error term

$$f(\varepsilon) := \int_0^{+\infty} f_u(u;\lambda) f_v\left(u+\varepsilon;\sigma^2\right) du =$$

$$= \frac{\sigma}{\lambda^2+\sigma^2} \frac{e^{-\frac{\varepsilon^2}{2\sigma^2}}}{\sqrt{2\pi}} \left(1 - \sqrt{\frac{\lambda^2\sigma^2}{\lambda^2+\sigma^2}} \frac{\varepsilon}{\sigma^2} e^{\frac{\varepsilon^2\lambda^2}{2\sigma^2(\sigma^2+\lambda^2)}} \sqrt{\frac{\pi}{2}} \left(erf\left(\frac{-\varepsilon}{\sqrt{2}\sigma^2} \sqrt{\frac{\lambda^2\sigma^2}{\lambda^2+\sigma^2}} \right) + 1 \right) \right) =$$

$$= \frac{\sigma}{\lambda^2+\sigma^2} \frac{e^{-\frac{\varepsilon^2}{2\sigma^2}}}{\sqrt{2\pi}} \left(1 - \frac{\lambda\varepsilon}{\sigma\sqrt{\lambda^2+\sigma^2}} e^{\frac{\varepsilon^2\lambda^2}{2\sigma^2(\sigma^2+\lambda^2)}} \sqrt{\frac{\pi}{2}} \left(erf\left(-\frac{\varepsilon\lambda}{\sigma\sqrt{2\left(\lambda^2+\sigma^2\right)}} \right) + 1 \right) \right).$$

This last equality can also be written:

$$f(\varepsilon) = \frac{\sigma}{\lambda^2+\sigma^2}\frac{e^{-\frac{\varepsilon^2}{2\sigma^2}}}{\sqrt{2\pi}}\left(1-\frac{\lambda\varepsilon}{\sigma\sqrt{\lambda^2+\sigma^2}}e^{\frac{\varepsilon^2\lambda^2}{2\sigma^2(\sigma^2+\lambda^2)}}\sqrt{2\pi}\Phi\left(-\frac{\varepsilon\lambda}{\sigma\sqrt{\lambda^2+\sigma^2}}\right)\right),$$

where Φ is the cumulative distribution of a standard normal random variable. As done by Jondrow et al. (1982) for half-normal and exponential case, now we may calculate the conditional distribution, $f(u|\varepsilon)$, of u given ε. It is the ratio between $f_u(u;\lambda)f_v(u+\varepsilon;\sigma^2)$ and $f(\varepsilon)$. So, we find

$$f(u|\varepsilon) =$$

$$= \frac{u(\lambda^2+\sigma^2)}{\sigma^2\lambda^2}e^{-\frac{u^2(\lambda^2+\sigma^2)}{2\sigma^2\lambda^2}-\frac{u\varepsilon}{\sigma^2}}\left(1-\frac{\lambda\varepsilon}{\sigma\sqrt{\lambda^2+\sigma^2}}e^{\frac{\varepsilon^2\lambda^2}{2\sigma^2(\sigma^2+\lambda^2)}}\sqrt{2\pi}\Phi\left(-\frac{\varepsilon\lambda}{\sigma\sqrt{\lambda^2+\sigma^2}}\right)\right)^{-1}.$$

Then, we assume that there is available a sample of I observations, ε_1, $\varepsilon_2, ..., \varepsilon_I$. In this case, we can form the log-likelihood function

$$\ln L\left(\varepsilon|\lambda,\sigma^2\right) = I\ln\left(\frac{\sigma}{\lambda^2+\sigma^2}\frac{1}{\sqrt{2\pi}}\right)+$$

$$+\sum_{i=1}^{I}\ln\left(1-\frac{\lambda\varepsilon_i}{\sigma\sqrt{\lambda^2+\sigma^2}}e^{\frac{\varepsilon_i^2\lambda^2}{2\sigma^2(\sigma^2+\lambda^2)}}\sqrt{2\pi}\Phi\left(-\frac{\varepsilon_i\lambda}{\sigma\sqrt{\lambda^2+\sigma^2}}\right)\right)-\sum_{i=1}^{I}\frac{\varepsilon_i^2}{2\sigma^2}.$$

Therefore, in order to calculate the ML estimators of λ and σ, we suggest finding the optimizing values by utilizing a direct numerical method: according to us, such a procedure has to be preferred, if compared to other numerical techniques based on taking partial derivatives of the likelihood function.

Recall that, if technical inefficiency terms follow a half-normal distribution, that is

$$f_u(u;\lambda) = \frac{2}{\sqrt{2\pi}\lambda}e^{-\frac{u^2}{2\lambda^2}}, \qquad u \geq 0, \qquad \lambda > 0,$$

then, the log-likelihood function is (see Aigner et al., 1977 or Behr and Tente, 2008)

$$\ln L\left(\varepsilon|\lambda,\sigma^2\right) =$$

$$= I\ln\left(\sqrt{\frac{2}{\pi}}\right)+I\ln\left(\frac{1}{\sqrt{\lambda^2+\sigma^2}}\right)+\sum_{i=1}^{I}\ln\left[1-\Phi\left(\frac{\lambda\varepsilon_i\sigma^{-1}}{\sqrt{\lambda^2+\sigma^2}}\right)\right]-\frac{\sum_{i=1}^{I}\varepsilon_i^2}{2(\lambda^2+\sigma^2)}.$$

Furthermore, when technical inefficiency is exponentially distributed, that is

$$f_u(u;\lambda) = \frac{1}{\lambda}e^{-\frac{u}{\lambda}}, \qquad u \geq 0, \qquad \lambda > 0$$

the relative log-likelihood is (Behr and Tente, 2008)

$$\ln L(\varepsilon|\lambda,\sigma^2) = -I\ln(\lambda)+\frac{I\sigma^2}{2\lambda^2}+\sum_{i=1}^{I}\ln\Phi\left(-\frac{\varepsilon_i}{\sigma}-\frac{\sigma}{\lambda}\right)+\frac{\sum_{i=1}^{I}\varepsilon_i}{\lambda}.$$

3 Monte Carlo simulations

We have performed the following Monte Carlo experiments: for given values of the parameters λ and σ, we have generated N samples

$$\{\varepsilon^{(n)}{}_i = -u^{(n)}{}_i + v^{(n)}{}_i,\ i \in \{1, 2, ..., I\},\ n \in \{1, 2, ..., N\}\},$$

whose dimensionality is I. For any $n \in \{1, 2, ..., N\}$, the sample $\{\varepsilon^{(n)}{}_i = -u^{(n)}{}_i + v^{(n)}{}_i,\ i \in \{1, 2, ..., I\}\}$, meets the assumptions (1-5) made in Paragraph 2. Therefore, $u^{(n)}{}_i \sim$ Rayleigh(λ) and $v^{(n)}{}_i \sim$ N(0, σ^2). Then, for any $n \in \{1, 2, ..., N\}$, we have estimated the two error parameters of the sample $\{\varepsilon^{(n)}{}_i = -u^{(n)}{}_i + v^{(n)}{}_i,\ i \in \{1, 2, ..., I\}\}$, by using the ML method. Moreover, we have repeated the experiments by considering the other two cases: $u^{(n)}{}_i \sim \mathrm{N}^+(0, \lambda^2)$ and $u^{(n)}{}_i \sim$ Exp(λ) ($i \in \{1, 2, ..., I\}, n \in \{1, 2, ..., N\}$). So doing, for any value of λ and σ, we have obtained the mean values and the standard deviations of the ML estimators. In order to perform this Monte Carlo analysis, we have utilized the Software R. We have reported the results of the simulations in Tables 1-2.

Table 1: Calculus of the mean values of the ML estimators, for given values of λ and σ, by using Monte Carlo procedure ($I = 50$, number of simulations = 1000).

	λ	σ	Mean value of σ_{ML}			Mean value of λ_{ML}		
$f_u(u;\lambda)$			Rayleigh(λ)	N$^+$(0,λ^2)	Exp(λ)	Rayleigh(λ)	N$^+$(0,λ^2)	Exp(λ)
	4	3	2.8735	2.9205	2.9595	4.0190	3.9815	3.9690
	3	4	3.9110	3.9175	3.9380	3.0155	2.9895	2.9895
	4	4	3.8915	3.8785	3.9340	3.9970	3.9845	3.9935

Table 2: Calculus of the standard deviations of the ML estimators, for given values of λ and σ, by using Monte Carlo procedure ($I = 50$, number of simulations = 1000).

	λ	σ	Standard deviation of σ_{ML}			Standard deviation of λ_{ML}		
$f_u(u;\lambda)$			Rayleigh(λ)	N$^+$(0,λ^2)	Exp(λ)	Rayleigh(λ)	N$^+$(0,λ^2)	Exp(λ)
	4	3	0.5772	0.5611	0.5693	0.4682	0.6896	0.6830
	3	4	0.5204	0.5436	0.6189	0.5132	0.7755	0.6905
	4	4	0.6397	0.5912	0.6785	0.5681	0.8542	0.8105

We observe that all ML estimators are distort in order to estimate noise parameter. On the contrary, they are correct when regarded as estimators of technical inefficiency. Moreover, when technical inefficiency components follow a Rayleigh distribution, the respective ML estimator appears to have a minor variance.

4 MOM estimators

Consider the equality

$$E(\varepsilon) = E(-u) = \lambda\sqrt{\frac{\pi}{2}}$$

and recall that, in a Rayleigh distribution with parameter λ, its second and third order central moments are equal, respectively, to $(4-\pi)\lambda^2/2$ and $\sqrt{\pi/2}(\pi-3)\lambda^3$. Furthermore, denote by m_i the i-th order moment of ε. In this case, following the considerations in Behr and Tente (2008) and adapting them to a Rayleigh distribution, we have

$$m_2 = \sigma^2 + \frac{4-\pi}{2}\lambda^2$$

$$m_3 = \sqrt{\frac{\pi}{2}}(\pi-3)\lambda^3.$$

From these equalities, we deduce the MOM estimators:

$$\lambda = \sqrt[6]{\frac{2m_3^2}{\pi(\pi-3)^2}}$$

$$\sigma = \sqrt{m_2 - \frac{4-\pi}{2}\lambda^2}.$$

Therefore, we have performed other computer experiments: we have generated a set of simulated values for ε as above performed. Furthermore, in any simulation, we have estimated the symmetric disturbance and technical inefficiency, by applying both ML and MOM techniques. We have resumed the relative results in Tables 3-4. Substantially, these last two tables confirm what stated in Tables 1-2. Moreover, we note that the ML estimators are less distort than MOM estimators. This is a well note result in the case of a single distribution function ($\lambda = 0$ or $\sigma = 0$). Furthermore, Table 3 tells us that, as I diverges, the ML estimators become correct also relatively to normal error component. Obviously, also MOM estimators are asymptotically correct, but, in this last case, the bias converges to zero more slowly.

5 Application

Now, we consider a dataset from Baten et al. (2009). They modelled inefficiency by the relation (Battese and Coelli, 1995)

$$u_{it} = \delta + \delta_1 z_{1it} + \delta_2 z_{2it} + \delta_3 z_{3it} + \delta_4 z_{4it} + \delta_5 z_{5it} + \tau_{it},$$

Table 3: Calculus of the mean values of various estimators, for given values of I, by using Monte Carlo procedure ($\lambda = 3,\ \sigma = 4$, number of simulations = 1000).

I	Mean value of estimated σ				Mean value of estimated λ			
	ML			MOM	ML			MOM
	Rayleigh	Half−normal	Expon.		Rayleigh	Half−normal	Expon.	
5	2.9750	3.1365	3.1540	2.1583	3.0575	3.0605	3.0070	4.8180
20	3.7660	3.7220	3.8290	2.6729	2.9645	3.0485	2.9605	4.9199
200	3.9850	3.9770	3.9900	3.5450	3.0055	2.9965	3.0000	3.8121

Table 4: Calculus of the standard deviations of various estimators, for given values of I, by using Monte Carlo procedure ($\lambda = 3,\ \sigma = 4$, number of simulations = 1000).

I	Standard deviation of estimated σ				Standard deviation of estimated λ			
	ML			MOM	ML			MOM
	Rayleigh	Half−normal	Expon.		Rayleigh	Half−normal	Expon.	
5	1.5841	1.5677	1.6668	1.1227	1.4235	1.7651	1.7451	2.3351
20	0.8233	0.8790	0.9858	1.0494	0.8022	1.1903	1.0223	1.7717
200	0.2929	0.2889	0.3285	0.5992	0.2970	0.4053	0.3715	1.1668

where z_{1it}, z_{2it}, z_{3it}, z_{4it} and z_{5it} are explanatory variables varying by region and time. Moreover, δ, δ_1, δ_2, δ_3, δ_4 and δ_5 are constant parameters and τ_{it} is a truncated normal random variable. So, the article reports the values of technical efficiency of the tea industry in seven regions of Bangladesh from 1990 to 2004 (Table 5).

First of all, we have calculated the inefficiency values by using the relation

$$u = -\log(TE),$$

where TE is the technical efficiency. Therefore, for any year, we have estimated noise and inefficiency parameters by various methods. In other words, we have let $N = 15$, $I = 7$ and $\sigma = -u$, where u are the values of regional inefficiencies in the specific year. We expect that, if the inefficiency values follow a given theoretical distribution, then the corresponding noise estimation is near to zero for any year. In fact, in half-normal and Rayleigh-ML case, $\hat{\sigma}$ is always relatively small with respect to $\hat{\lambda}$ (Table 6). Furthermore, in the Rayleigh-ML case, both the mean and the variance of $\hat{\sigma}$ are smaller when compared to other cases.

Table 5: Wise Mean Efficiency of Yield for various regions in Bangladesh, 1990-2004.

year	North Sylhet	Jury valley	Lungla	Manu-doloi	Balisera	Luskerpore	Ctg. dis
1990	0.39	0.46	0.42	0.59	0.43	0.86	0.37
1991	0.43	0.52	0.41	0.67	0.73	0.71	0.38
1992	0.37	0.49	0.29	0.60	0.67	0.60	0.33
1993	0.34	0.42	0.32	0.56	0.66	0.60	0.33
1994	0.37	0.55	0.37	0.65	0.76	0.69	0.38
1995	0.30	0.47	0.31	0.52	0.58	0.53	0.29
1996	0.36	0.54	0.38	0.60	0.70	0.60	0.39
1997	0.31	0.44	0.29	0.47	0.49	0.50	0.31
1998	0.57	0.83	0.56	0.89	0.91	0.92	0.54
1999	0.31	0.54	0.37	0.59	0.66	0.57	0.35
2000	0.39	0.52	0.39	0.60	0.72	0.50	0.47
2001	0.42	0.54	0.37	0.65	0.67	0.49	0.58
2002	0.35	0.44	0.32	0.57	0.66	0.43	0.40
2003	0.40	0.52	0.36	0.67	0.77	0.50	0.44
2004	0.36	0.47	0.33	0.60	0.70	0.45	0.42

6 Conclusions

In this paper, we have presented an alternative approach to the stochastic frontier analysis. The main contribution is the implementation of a numerical procedure for calculating the ML estimators of the total error parameters, when the one-tailed terms follow a Rayleigh distribution. For this purpose, we have derived the probability density function, of total error, in a closed form. In order to develop this model, we have assumed that the density distribution of technical inefficiency is zero at the origin. We have compared this Rayleigh model to half-normal and exponential case. We have verified that all the ML estimators are correct, in order to estimate technical inefficiency. Furthermore, in the Rayleigh case, the relative variance is minor. At the end, we have derived the MOM estimators: they are distort and have a greater variance than ML estimators. For this reason, we do not recommend the use of MOM when the data set of error observations is not large.

Table 6: Estimation of inefficiency and noise parameters for regions in Bangladesh, 1990-2004.

	ML Rayleigh		ML half − normal		ML exponential		MOM	
	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\lambda}$
1990	0.028	0.552	0.004	0.780	0.056	0.724	0.196	0.514
1991	0.028	0.480	0.040	0.680	0.164	0.592	0.246	0.116
1992	0.036	0.592	0.016	0.840	0.236	0.716	0.220	0.325
1993	0.032	0.608	0.072	0.860	0.260	0.736	0.222	0.275
1994	0.028	0.512	0.004	0.724	0.156	0.628	0.269	0.181
1995	0.036	0.656	0.056	0.928	0.324	0.776	0.212	0.285
1996	0.028	0.528	0.056	0.744	0.236	0.632	0.218	0.180
1997	0.036	0.680	0.024	0.964	0.396	0.776	0.156	0.249
1998	0.020	0.280	0.004	0.396	0.032	0.316	0.157	0.262
1999	0.032	0.572	0.076	0.808	0.252	0.684	0.186	0.315
2000	0.028	0.508	0.068	0.716	0.276	0.588	0.121	0.253
2001	0.028	0.484	0.072	0.680	0.248	0.564	0.113	0.260
2002	0.032	0.604	0.068	0.852	0.332	0.700	0.117	0.316
2003	0.028	0.512	0.056	0.724	0.176	0.640	0.141	0.320
2004	0.032	0.576	0.028	0.812	0.292	0.676	0.139	0.312
Mean	0.0301	0.5429	0.0429	0.7672	0.2291	0.6540	0.1809	0.2775
St. Dev.	0.0042	0.0941	0.0273	0.1335	0.0988	0.1166	0.0497	0.0890

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